OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4769**

Statistics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Friday 18 June 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 6 9 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Option 1: Estimation

1 The random variable X has probability density function

$$f(x) = \frac{xe^{-x/\lambda}}{\lambda^2} \qquad (x > 0),$$

where λ is a parameter ($\lambda > 0$). $X_1, X_2, \ldots, X_n$ are n independent observations on X, and $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is their mean.

(i) Obtain $E(X)$ and deduce that $\hat{\lambda} = \frac{1}{2}\overline{X}$ is an unbiased estimator of λ. **[7]**

(ii) Obtain $\mathrm{Var}(\hat{\lambda})$. **[7]**

(iii) Explain why the results in parts **(i)** and **(ii)** indicate that $\hat{\lambda}$ is a good estimator of λ in large samples. **[2]**

(iv) Suppose that $n = 3$ and consider the alternative estimator

$$\tilde{\lambda} = \tfrac{1}{8}X_1 + \tfrac{1}{4}X_2 + \tfrac{1}{8}X_3.$$

Show that $\tilde{\lambda}$ is an unbiased estimator of λ. Find the relative efficiency of $\tilde{\lambda}$ compared with $\hat{\lambda}$. Which estimator do you prefer in this case? **[8]**

Option 2: Generating Functions

2 The random variable X has the Poisson distribution with parameter λ.

(i) Show that the probability generating function of X is $G(t) = e^{\lambda(t-1)}$. **[3]**

(ii) Hence obtain the mean μ and variance σ^2 of X. **[5]**

(iii) Write down the mean and variance of the random variable $Z = \dfrac{X - \mu}{\sigma}$. **[2]**

(iv) Write down the moment generating function of X. State the linear transformation result for moment generating functions and use it to show that the moment generating function of Z is

$$M_Z(\theta) = e^{f(\theta)} \quad \text{where } f(\theta) = \lambda\left(e^{\theta/\sqrt{\lambda}} - \frac{\theta}{\sqrt{\lambda}} - 1\right).$$ **[7]**

(v) Show that the limit of $M_Z(\theta)$ as $\lambda \to \infty$ is $e^{\theta^2/2}$. **[4]**

(vi) Explain briefly why this implies that the distribution of Z tends to $N(0, 1)$ as $\lambda \to \infty$. What does this imply about the distribution of X as $\lambda \to \infty$? **[3]**

Option 3: Inference

3 At a factory, two production lines are in use for making steel rods. A critical dimension is the diameter of a rod. For the first production line, it is assumed from experience that the diameters are Normally distributed with standard deviation 1.2 mm. For the second production line, it is assumed from experience that the diameters are Normally distributed with standard deviation 1.4 mm. It is desired to test whether the mean diameters for the two production lines, μ_1 and μ_2, are equal. A random sample of 8 rods is taken from the first production line and, independently, a random sample of 10 rods is taken from the second production line.

(i) Find the acceptance region for the customary test based on the Normal distribution for the null hypothesis $\mu_1 = \mu_2$, against the alternative hypothesis $\mu_1 \neq \mu_2$, at the 5% level of significance. **[6]**

(ii) The sample means are found to be 25.8 mm and 24.4 mm respectively. What is the result of the test? Provide a two-sided 99% confidence interval for $\mu_1 - \mu_2$. **[7]**

The production lines are modified so that the diameters may be assumed to be of equal (but unknown) variance. However, they may no longer be Normally distributed. A two-sided test of the equality of the population medians is required, at the 5% significance level.

(iii) The diameters in independent random samples of sizes 6 and 8 are as follows, in mm.

First production line	25.9	25.8	25.3	24.7	24.4	25.4		
Second production line	23.8	25.6	24.0	23.5	24.1	24.5	24.3	25.1

Use an appropriate procedure to carry out the test. **[11]**

[Question 4 is printed overleaf.]

Option 4: Design and Analysis of Experiments

4 At an agricultural research station, a trial is made of four varieties (A, B, C, D) of a certain crop in an experimental field. The varieties are grown on plots in the field and their yields are measured in a standard unit.

(i) It is at first thought that there may be a consistent trend in the natural fertility of the soil in the field from the west side to the east, though no other trends are known. Name an experimental design that should be used in these circumstances and give an example of an experimental layout. **[5]**

Initial analysis suggests that any natural fertility trend may in fact be ignored, so the data from the trial are analysed by one-way analysis of variance.

(ii) The usual model for one-way analysis of variance of the yields y_{ij} may be written as

$$y_{ij} = \mu + \alpha_i + e_{ij}$$

where the e_{ij} represent the experimental errors. Interpret the other terms in the model. State the usual distributional assumptions for the e_{ij}. **[7]**

(iii) The data for the yields are as follows, each variety having been used on 5 plots.

Variety			
A	B	C	D
12.3	14.2	14.1	13.6
11.9	13.1	13.2	12.8
12.8	13.1	14.6	13.3
12.2	12.5	13.7	14.3
13.5	12.7	13.4	13.8

$[\Sigma\Sigma y_{ij} = 265.1, \quad \Sigma\Sigma y_{ij}^2 = 3524.31.]$

Construct the usual one-way analysis of variance table and carry out the usual test, at the 5% significance level. Report briefly on your conclusions. **[12]**

GCE

MEI Mathematics

Advanced GCE **4769**

Statistics 4

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Question 1

	$f(x) = \dfrac{xe^{-x/\lambda}}{\lambda^2} \quad (x > 0)$	
(i)	$E(X) = \dfrac{1}{\lambda^2}\int_0^\infty x^2 e^{-x/\lambda}dx$ $= \dfrac{1}{\lambda^2}\left\{\left[-\lambda x^2 e^{-x/\lambda}\right]_0^\infty + \int_0^\infty \lambda.2xe^{-x/\lambda}dx\right\}$ $= \dfrac{1}{\lambda^2}\{[0-0]\} + 2\lambda.1 = 2\lambda$. $E(\bar{X}) = E(X) \quad \therefore E\left(\hat{\lambda}[= \tfrac{1}{2}\bar{X}]\right) = \lambda \quad \therefore \hat{\lambda}$ is unbiased.	M1 for integral for E(X) M1 for attempt to integrate by parts For second term: M1 for use of integral of pdf or for integr'g by parts again A1 M1 A1 E1 **[7]**
(ii)	$\text{Var}(\hat{\lambda}) = \dfrac{1}{4}\text{Var}(\bar{X}) = \dfrac{1}{4}\dfrac{\text{Var}(X)}{n}$ $E(X^2) = \dfrac{1}{\lambda^2}\int_0^\infty x^3 e^{-x/\lambda}dx$ $= \dfrac{1}{\lambda^2}\left\{\left[-\lambda x^3 e^{-x/\lambda}\right]_0^\infty + \int_0^\infty 3\lambda x^2 e^{-x/\lambda}dx\right\}$ $= \dfrac{1}{\lambda^2}\{[0-0]\} + 3\lambda E(X) = 6\lambda^2$. $\therefore \text{Var}(X) = E(X^2) - \{E(X)\}^2 = 6\lambda^2 - 4\lambda^2 = 2\lambda^2$. $\therefore \text{Var}(\hat{\lambda}) = \dfrac{\lambda^2}{2n}$.	M1 M1 for use of E(X^2) By parts M1 M1 for use of E(X) A1 for $6\lambda^2$ A1 A1 **[7]**
(iii)	Variance of $\hat{\lambda}$ becomes very small as n increases. It is unbiased and so becomes increasingly concentrated at the correct value λ.	E1 E1 **[2]**
(iv)	$E(\tilde{\lambda}) = \left(\tfrac{1}{8} + \tfrac{1}{4} + \tfrac{1}{8}\right)2\lambda = \lambda$. $\quad \therefore \tilde{\lambda}$ is unbiased. $\text{Var}(\tilde{\lambda}) = \left(\tfrac{1}{64} + \tfrac{1}{16} + \tfrac{1}{64}\right)2\lambda^2 = \tfrac{3}{16}\lambda^2$. $\therefore$ relative efficiency of $\tilde{\lambda}$ to $\hat{\lambda}$ is $\dfrac{\lambda^2/6}{3\lambda^2/16} = \dfrac{8}{9}$. **Special case.** If done as Var($\tilde{\lambda}$) / Var($\hat{\lambda}$), award 1 out of 2 for the second M1 and the A1 in the scheme. So $\hat{\lambda}$ is preferred.	$E(\tilde{\lambda})$: B1; "unbiased": E1 M1 A1 M1 any comparison of variances M1 correct comparison A1 for 8/9 [Note. This M1M1A1 is allowable in full as FT if everything is plausible.] E1 (FT from above) **[8]**

Question 2

(i)	$G(t) = E(t^X) = \sum_{x=0}^{\infty} \frac{e^{-\lambda}(\lambda t)^x}{x!}$ **[M1]** $= e^{-\lambda}\left(1 + \lambda t + \frac{\lambda^2 t^2}{2!} + ...\right)$ **[A1]** $= e^{-\lambda} e^{\lambda t} = e^{\lambda(t-1)}$ **[A1]** **[Allow omission of previous A1 step and write-down of this for A2 provided opening M1 has been earned (NB answer is given)]**	**[3]**
(ii)	Mean = G'(1) $G'(t) = \lambda e^{\lambda(t-1)}$ **[M1]** G'(1) = λ **[A1]** Variance = G''(1) + mean – mean^2 $G''(t) = \lambda^2 e^{\lambda(t-1)}$ **[M1]** G''(1) = λ^2 **[A1]** $\therefore$ variance = $\lambda^2 + \lambda - \lambda^2 = \lambda$ **[A1]**	**[5]**
(iii)	$Z = \frac{X - \mu}{\sigma}$: mean 0 **[B1]** variance 1 **[B1]**	**[2]**
(iv)	Mgf of X is $M(\theta) = G(e^\theta) = e^{\lambda(e^\theta - 1)}$ **[B1]** Linear transformation result is $M_{aX+b}(\theta) = e^{b\theta} M_X(a\theta)$ **[B2 if fully correct, any equivalent form. Allow B1 if either factor correct.]** Use with $a = \frac{1}{\sigma} = \frac{1}{\sqrt{\lambda}}$ and $b = -\frac{\mu}{\sigma} = -\sqrt{\lambda}$ **[M1]** $M_Z(\theta) = e^{-\sqrt{\lambda}\theta} e^{\lambda\left(e^{\theta/\sqrt{\lambda}} - 1\right)} = e^{\lambda\left(e^{\theta/\sqrt{\lambda}} - \frac{\theta}{\sqrt{\lambda}} - 1\right)}$ **[A1] [A1] [A1] [NB answer is given]**	**[7]**
(v)	Consider $\lambda\left(e^{\theta/\sqrt{\lambda}} - \frac{\theta}{\sqrt{\lambda}} - 1\right) = \lambda\left(1 + \frac{\theta}{\sqrt{\lambda}} + \frac{\theta^2}{2!\lambda} + \frac{\theta^3}{3!\lambda^{3/2}} + ... - \frac{\theta}{\sqrt{\lambda}} - 1\right)$ **[M1]** $= \frac{\theta^2}{2}$ + terms in $\lambda^{-1/2}, \lambda^{-1}, \lambda^{-3/2}, ...$ **[A1]** $\rightarrow \frac{\theta^2}{2}$ as $\lambda \rightarrow \infty$ **[M1]** **[some explanation required]** $\therefore M_Z(\theta) \rightarrow e^{\theta^2/2}$ as $\lambda \rightarrow \infty$ **[A1] [answer given]**	**[4]**
(vi)	$e^{\theta^2/2}$ is the mgf of N(0, 1) **[E1]** , and the relationship between distributions and their mgfs is unique **[E1]** . "Unstandardising", X tends to N(μ, σ^2) i.e. N(λ, λ) **[B1, parameters must be given]**.	**[3]**

Question 3

(i)	H_0 is accepted if –1.96 < value of test statistic < 1.96 i.e. if $-1.96 < \dfrac{(\bar{x}_1 - \bar{x}_2) - (0)}{\sqrt{\dfrac{1.2^2}{8} + \dfrac{1.4^2}{10}}} < 1.96$ i.e. if $-1.96 \times 0.6132 < \bar{x}_1 - \bar{x}_2 < 1.96 \times 0.6132$ i.e. if $-1.20(18) < \bar{x}_1 - \bar{x}_2 < 1.20(18)$ **Note.** Use of $\mu_1 - \mu_2$ instead of $\bar{x}_1 - \bar{x}_2$ can score M1 B1 M0 M1 A0 A0.	M1 double inequality B1 1.96 M1 num^r of test statistic M1 den^r of test statistic A1 A1 **Special case.** Allow 1 out of 2 of the A1 marks if 1.645 used provided all 3 M marks have been earned. **[6]**
(ii)	$\bar{x}_1 - \bar{x}_2 = 1.4$ which is outside the acceptance region so H_0 is rejected. CI for $\mu_1 - \mu_2$: 1.4 ± (2.576 × 0.6132), i.e. 1.4 ± 1.5796, i.e. (–0.18 [–0.1796] , 2.97[96])	B1 FT if wrong M1 [FT can's acceptance region if reasonable] E1 M1 for 1.4 B1 for 2.576 M1 for 0.6132 A1 cao for interval **[7]**
(iii)	Wilcoxon rank sum test (or Mann-Whitney form of test) Ranks are: First 14 13 10 8 6 11 Second 2 12 3 1 4 7 5 9 W = 14 + 13 + 10 + 8 + 6 + 11 = 62 [or 8 + 8 + 7 + 7 + 6 + 5 = 41 if M-W used] Refer to $W_{6,8}$ [or $MW_{6,8}$] tables. Lower 2½% critical point is 29 [or 8 if M-W used]. Consideration of upper 2½% point is also needed. Eg: by using symmetry about mean of $\left(\frac{1}{2}\times 6\times 8\right)+\left(\frac{1}{2}\times 6\times 7\right)$ = 45, critical point is 61. [For M-W: mean is $\frac{1}{2}\times 6\times 8$ = 24, hence critical point is 40.] Result is significant. Seems (population) medians may not be assumed equal.	M1 M1 Combined ranking A1 Correct [allow up to 2 errors; FT provided M1 earned] B1 M1 No FT if wrong A1 **Special case 1.** If M1 for $W_{6,8}$ has not been awarded (likely to be because rank sum 43 has been used, which should be referred to $W_{8,6}$), the next two M1 marks can be earned but *nothing beyond them*. M1 M1 for any correct method A1 if 61 correct E1, E1 **Special case 2** (does not apply if Special Case 1 has been invoked). These 2 marks may be earned even if only 1 or 2 of the preceding 3 have been earned. **[11]**

Question 4

(i) Randomised blocks	B1
Eg:-	
<table><tr><td>WEST</td><td>D A C B</td><td>C B A D</td><td>D C A B</td><td>EAST</td></tr></table>	
Plots in strips (blocks)	M1
correctly aligned w.r.t. fertility trend.	E1
Each letter occurs at least once in each block	M1
in a random arrangement.	E1 **[5]**
(ii) μ = population **[B1]** grand mean for whole experiment **[B1]** α_i = population **[B1]** mean amount by which the *i*th treatment differs from μ **[B1]**	4 marks, as shown
e_{ij} ~ ind N [**B1**, accept "uncorrelated"] (0 **[B1]** , σ^2 **[B1]**)	3 marks, as shown **[7]**
(ii) Totals are 62.7 65.6 69.0 67.8 all from samples of size 5	
Grand total 265.1 "Correction factor" CF = $265.1^2/20$ = 3513.9005	
Total SS = 3524.31 – CF = 10.4095 Between varieties SS = $\frac{62.7^2}{5}+\frac{65.6^2}{5}+\frac{69.0^2}{5}+\frac{67.8^2}{5}$ – CF = 3518.498 – CF = 4.5975 Residual SS (by subtraction) = 10.4095 – 4.5975 = 5.8120	M1 for attempt to form three sums of squares. M1 for correct method for any two. A1 if each calculated SS is correct.
<table><tr><th>Source of variation</th><th>SS</th><th>df</th><th>MS **[M1]**</th><th>MS ratio **[M1]**</th></tr><tr><td>Between varieties</td><td>4.5975</td><td>3 **[B1]**</td><td>1.5325</td><td>4.22 **[A1 cao]**</td></tr><tr><td>Residual</td><td>5.8120</td><td>16 **[B1]**</td><td>0.36325</td><td></td></tr><tr><td>Total</td><td>10.4095</td><td>19</td><td></td><td></td></tr></table>	5 marks within the table, as shown
Refer MS ratio to $F_{3,16}$.	M1 No FT if wrong
Upper 5% point is 3.24. Significant. Seems the mean yields of the varieties are not all the same.	A1 No FT if wrong E1 E1 **[12]**

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

Chief Examiners' Report

In this series, as always, the Principal Examiners' reports have tried to give teachers information to help them to evaluate the work of their students in the context of the strengths and weaknesses of the overall entry.

Some weaknesses are commonly mentioned: poor recognition and use of 'technical' language and notation, failure to present methods or reasons clearly and failure to set out work clearly.

Any candidate who does not know the meaning of technical words or notation given in the specification is at a great disadvantage. This is obviously the case when this lack of knowledge prevents the candidate from completely understanding what is required but also, poor or inaccurate use of technical terms or notation can impair a candidate's attempt to comment on an answer or explain a method.

Almost all solutions should include a clear indication of the method used. The rubric for each paper advises candidates that 'an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used'. Of course, when candidates are asked to establish a *given* answer, the detail required may be much greater that when the answer is not known.

Good, clear (and compact) display of working helps a candidate produce a coherent argument and reduces the chance of 'slips'. Candidates of all levels of ability can benefit from presenting their work and ideas well and there is often an association between good layout and high quality of work. It is to be hoped that the introduction of Printed Answer Books will encourage candidates to consider more carefully their setting out of solutions.

There are three matters that have been raised about how candidates should use the Printed Answer Books (that will be scanned). The first is that they should put their answers in the correct sections; the second is that they should not try to erase writing or drawing but should cross it out – the scanning process is sensitive and copies the faint images and marks that often are left after attempts at erasure. Finally it should be noted that the use of additional answer sheets should be unusual, and that sheets of rough working should not be handed in.

Note on accuracy in Statistics modules

The Principal Examiners' reports that follow discuss the candidates' performances on the individual modules. There is one matter that should be discussed in a general way as it applies to all the statistics modules. This is in respect of arithmetical accuracy in intermediate working and in quotation of final answers. Please note that these remarks are specific to the *statistics* modules; they do not necessarily apply to other modules, where it may be natural for somewhat different criteria to be appropriate.

Most candidates are sensible in their arithmetical work, but there is some unease as to exactly what level of accuracy the examiners are expecting. There is no general answer to this! The standard rubric for all the papers sums the situation up by including "final answers should be given to a degree of accuracy appropriate to the context". Three significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting from Normal tables, *some* evidence of interpolation is generally expected and so quotation to four decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice.

Talking now in general terms, the examiners always exercise sensible discretion in cases of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, a candidate is likely to lose an Accuracy mark; but if 4 significant figures are given, there would normally be no penalty. Likewise, answers which are slightly deviant from what is expected in a very minor manner are not penalised (for example, a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected). However, there are increasing numbers of cases where candidates give answers which are *grossly* over- or under-specified, such as insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happens to come off the candidate's calculator. **Such gross over-specification indicates a lack of appreciation of the nature of statistical work and, with effect from the January 2011 examinations, will be penalised by withholding of associated Accuracy marks.**

Candidates must however always be aware of the dangers of premature rounding if there are several steps in a calculation. If, say, a final answer is desired that is correct to 3 decimal places, this can in no way be guaranteed if only 3 decimal places are used in intermediate steps; indeed, it may not be safe to carry out the intermediate work even to 4 decimal places. The issue of over-specification may arise for the final answer but not for intermediate stages of the working.

It is worth repeating that most candidates act sensibly in all these respects, but it is hoped that this note may help those who are perhaps a little less confident in how to proceed.

4769 Statistics 4

General comments
There were 31 candidates from 12 centres (plus 4 more centres, each of whose candidates were absent). While this is obviously a small entry, it is pleasing that it is holding up. It is only slightly down on last year and is a noticeable and welcome increase from the year before last.

It is also pleasing to report that there was much very good work – for the paper as a whole and for each individual question. Sadly there was also some poor work, but the good work was very much in the majority.

As usual, the paper consisted of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted, and all candidates obeyed this. Question 4, on design and analysis of experiments, was very much the least popular question, with only a handful of attempts – not a feature that has occurred in previous years. The other three questions were equally popular.

Comments on individual questions

1) This was on the "estimation" option. It was mainly about investigating two unbiased estimators and comparing their variances.

The question involved integration of functions of the form $x^n e^{-x}$ for fairly small integer values of *n*. Candidates, even those who were successful, seemed to make fairly heavy weather of this. Many candidates did much more work than they needed by not seeing that the integration by parts in part (i) of the question re-created the pdf of the original random variable whose integral could be written down as 1; and then again by not seeing that the integration by parts in part (ii) re-created the integral that had already been found in part (i).

Part (iii) sought an explanation of two desirable features of the estimator – its variance becomes very small as *n* increases and so, being unbiased, it becomes increasingly concentrated at the correct value of the parameter. Most explanations more-or-less made these points, but sometimes not very securely. It was pleasing to see that some candidates were familiar with the correct technical term "consistent".

The second estimator was introduced in part (iv). Candidates generally knew that it should be compared with the first in terms of their variances. Some candidates had the relative efficiency definition "upside down", though they still generally knew how to use the result.

2) This was on the "generating functions" option and explored the Normal approximation to the Poisson distribution.

Many intermediate answers were given in this question, partly for the comfort of candidates as they successfully worked through it and partly so that candidates who could not derive a result could nevertheless use it in the sequel. The usual point has to be made that, where an answer is given, candidates have to be *convincing* in their derivations of it. In fact most candidates were, except in the limiting result in part (v).

Part (i) asked for the probability generating function of the Poisson distribution and part (ii) sought derivation of the mean and variance. These parts were usually done without difficulty.

In part (iii), it was remarkable that many candidates were unable *simply* to write down the mean (zero) and variance (one) of the standardised variable.

In part (iv), most candidates knew that the moment generating function is of the same form as the probability generating function, merely with a change of variable; and most candidates could write down the linear transformation result without much ado. Using this to obtain the moment generating function of the "standardised Poisson" required some care in algebra, but mostly this was done successfully.

Part (v) was where more than a few candidates had problems, not knowing how to handle the limiting process. However, several candidates were fully successful here.

In part (vi), candidates mostly realised the importance of the uniqueness of the relation between a distribution and its moment generating function, though this was not always explicitly stated. The "unstandardising" was usually understood, except that several candidates did not seem to appreciate that the unstandardised mean and variance were both λ (the parameter of the original Poisson distribution) – which is a key feature of this Normal approximation.

3) This question was on the "inference" option.

Part (i) asked, fairly formally, for the acceptance region to be set up for an unpaired Normal test. Many candidates knew what to do and correctly obtained it (even if not necessarily following the explicit steps that are set out in the published mark scheme), but there were a number of errors here. The worst error, and it is a serious mistake and especially sad to see in candidates working at Statistics 4 level, was to express the acceptance region as a double inequality on the difference in the *population* means, not the sample means. The logical absurdity of this seemed to escape these candidates. Other rather bad errors occurred in the denominator of the test statistic, where a number of incorrect forms appeared.

Part (ii) opened by giving a numerical value of the difference in sample means and asking what was the result of the test. All that had to be done here was to note that this value was outside the acceptance region and therefore the null hypothesis is rejected. While several candidates did this, many actually performed the full significance test – this of course leads to the correct answer but, as well as being a waste of time and effort, suggests that these candidates did not understand the force of the general concept of an acceptance region.

Part (ii) concluded by asking for a confidence interval. This was usually done well.

Part (iii) moved on to the Wilcoxon rank sum test. A few candidates made the error of thinking that "the smaller sample" (from which the rank sum is obtained) means the one which contains the numerically smaller values rather than the one whose size is smallest. This matters, because published tables are always drawn up on the basis of the rank sum coming from the sample of smaller size. It was however particularly pleasing that most candidates realised that, in this case, they needed to consider the upper tail points as well as the tabulated lower tail points in order to establish whether the result was significant; and, further, most of these candidates used a valid method based on symmetry to obtain the upper point.

4) This was on the "design and analysis of experiments" option. As already mentioned, there were very few attempts at this question, so these notes are only very brief so as to avoid the danger of accidental identification of individual candidates.

Candidates knew that the required design was randomised blocks and gave good descriptions of this design. They were generally sound with the modelling, and they were able to construct and interpret the required one-way analysis of variance.